

Baryon-Baryon Interactions from the Lattice

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Abstract. I discuss the latest progress in calculating baryon-baryon interactions with lattice QCD. The latest results from the NPLQCD collaboration for nucleon-nucleon scattering along with preliminary calculations of hyperon-nucleon scattering are presented.

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1 Introduction

Perhaps the greatest challenge facing those of us working in the area of strong interaction physics is to be able to rigorously compute the properties and interactions of nuclei. The many decades of theoretical and experimental investigations in nuclear physics have, in many instances, provided a very precise phenomenology of the strong interactions in the non-perturbative regime. However, at this point in time we have little understanding of much of this phenomenology in terms of the underlying theory of the strong interactions, Quantum Chromo Dynamics (QCD).

The ultimate goal is to be able to rigorously compute the properties and interactions of nuclei from QCD. This includes determining how the structure of nuclei depend upon the fundamental constants of nature. Any nuclear observable is essentially a function of only five constants, the length scale of the strong interactions, Λ_{QCD} , the quark masses, m_u , m_d , m_s , and the electromagnetic coupling, α_e (at low energies the dependence upon the top, bottom and charm quarks masses is encapsulated in Λ_{QCD}). Perhaps as important, we would then be in the position to reliably compute quantities that cannot be accessed, either directly or indirectly, by experiment.

The only way to rigorously compute strong-interaction quantities in the nonperturbative regime is with lattice QCD. One starts with the QCD Lagrange density and performs a Monte-Carlo evaluation of Euclidean space Green functions directly from the path integral. To perform such an evaluation, space-time is latticized and computations are performed in a finite volume, at finite lattice spacing, and at this point in time, with quark masses that are larger than the physical quark masses. To compute any given quantity, contractions are performed in which the valence quarks that propagate on any given gauge-field configuration are “tied together”. For simple processes such as nucleon-nucleon scattering, such contractions do not require significant computer time compared with lattice or propagator generation. However, as one explores

processes involving more hadrons, the number of contractions grows rapidly (for a nucleus with atomic number A , charge Z and strangeness S , the number of contractions is $(A + Z)!(2A - Z - S)!S!^1$), and a direct lattice QCD calculation of the properties of a large nucleus is quite impractical simply due to the computational time required. The way to proceed is to establish a small number of effective theories, each of which have well-defined expansion parameters and can be shown to be the most general form consistent with the symmetries of QCD. Each theory must provide a complete description of nuclei over some range of atomic number. Calculations in two “adjacent” theories are performed for a range of atomic numbers for which both theories converge. One then matches coefficients in one EFT to the calculations in the other EFT or to the lattice, and thereby one can make an indirect, but rigorous, connection between QCD and nuclei. It appears that four different matchings are required:

1. **Lattice QCD.** Lattice QCD calculations of the properties of the very lightest nuclei will be possible at some point in the not so distant future [1]. Calculations for $A \leq 4$ as a function of the light-quark masses, would uniquely define the interactions between nucleons up to and including the four-body operators. Depending on the desired precision, one could possibly imagine calculations up to $A \sim 8$.

The chiral potentials and interactions have been determined out to the order where four-body interactions contribute [2]. As with the EFT constructions in the meson-sector and single-nucleon sector, the number of counterterms proliferates with increasing order in the expansion, and at some order one loses rigorous predictive power without external input. Lattice QCD will make model independent determinations of these counterterms.

¹ Notice that the number of contractions in hypernuclei containing a small number of hyperons is less than in ordinary non-strange nuclei.

2. **Exact Many-Body Methods.** During the past decade one has seen remarkable progress in the calculation of nuclear properties using Green Function Monte-Carlo (GFMC) with the AV_{18} -potential (e.g. Ref. [3]) and also the No-Core Shell Model (NCSM) (e.g. Ref. [4]) using chiral potentials. Starting with the chiral potentials, which are the most general interactions between nucleons consistent with QCD, one would calculate the properties of nuclei as a function of all the parameters in the chiral potentials out to some given order in the chiral expansion. A comparison between such calculations and lattice QCD calculations will determine these parameters to some level of precision. These parameters can then be used in the calculation of nuclear properties up to atomic numbers $A \sim 20 - 30$. The computer time for these many-body theories suffers from the same $\sim (A!)^2$ blow-up that lattice QCD does, and for a sufficiently large nucleus, such calculations become impractical.

Another recent development that shows exceptional promise is the latticization of the chiral effective field theories [5, 6, 7, 8, 9]. This should provide a model-independent calculation of nuclear processes once matched to lattice QCD calculations.

3. **Coupled Cluster Calculations.** In order to move to larger nuclei, $A \lesssim 100$ a technique that has shown promise is to implement a coupled-clusters expansion (e.g. Ref. [10]). One uses the same chiral potential that will have been matched to lattice QCD calculations, and then performs a diagonalization of the nuclear Hamiltonian, after truncating the cluster expansion, which itself contains arbitrary coefficients. The results of these calculations will be matched to those of the NCSM or GFMC for $A \sim 20 - 30$ to determine the arbitrary coefficients. This method is unlikely to be practical for very large atomic numbers.
4. **Very Large Nuclei and Density Functional Theory (Maybe)** To complete the periodic table one needs to have an effective theory that is valid for very large nuclei and nuclear matter. A candidate that has received recent attention is Density Function Theory (DFT) (e.g. Refs. [11, 12]). It remains to be seen if this is in fact a viable candidate. There is reason to hope that this will be useful because there is clearly a density expansion in large nuclei with a power-counting that is consistent with the Naive Dimensional Analysis (NDA) of Georgi and Manohar [13]. The application of DFT to large nuclei is presently the least rigorously developed component of this program.

The latticized chiral theory mentioned previously can also be applied to the infinite nuclear matter problem. This work is still in the very earliest stages of exploration, but this looks promising [7].

2 Lattice QCD Calculations of Single Nucleon Properties

While the lattice QCD calculations have historically focused on the meson sector, both the properties of single

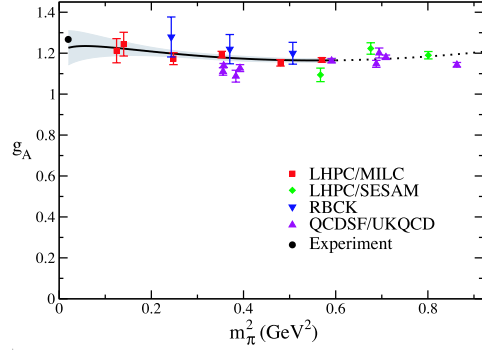


Fig. 1. g_A as a function of m_π^2 . This figure is reproduced with the permission of LHPC.

mesons and the interactions between two mesons, primarily due to limitations in computational power, the last few years has seen an increasing number of precise calculations of the properties of nucleons at the available pion masses. I wish to show two that will be of interest to the participants of this meeting.

2.1 The Matrix Element of the Axial Current

The matrix element of the axial current in the nucleon, g_A , is a fundamental quantity in nuclear physics as it is related to the strength of the long-range part of the nucleon-nucleon interaction via PCAC. During the last year LHPC [14] has for the first time calculated g_A at small enough pion masses where Heavy Baryon Chiral Perturbation Theory (HB χ PT) should converge. The results of this calculation, previous calculations and the physical value, are shown in Fig. 1. Also shown is the chiral extrapolation along with its uncertainty (the shaded region). The physical value lies within the range predicted by chiral extrapolation of the lattice calculations.

2.2 The Neutron-Proton Mass Difference

During the last year it was realized that one could use isospin-symmetric lattices to compute isospin-breaking quantities to NLO in the chiral expansion [15]. This is achieved by performing partially-quenched (unphysical) calculations of the nucleon mass [16] in which the valence quark masses differ from the sea-quark masses (those of the configurations) and determining counterterms in the partially-quenched chiral Lagrangian. The NPLQCD collaboration found that, in the absence of electromagnetic interactions, the neutron-proton mass difference at the physical value of the quark masses is

$$M_n - M_p|^{m_d=m_u} = +2.26 \pm 0.57 \pm 0.42 \pm 0.10 \text{ MeV} \quad (1)$$

which is to be compared with estimates derived from the Cottingham sum-rule of $M_n - M_p|_{\text{physical}}^{m_d=m_u} = +2.05 \pm 0.3$. We see that the lattice determination is consistent with what is found in nature. The lattice calculation is expected to become significantly more precise during the next year.

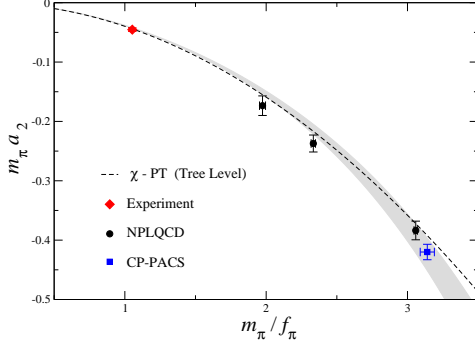


Fig. 2. The $\pi\pi$ scattering length in the $I = 2$ channel as a function of m_π^2/f_π^2 . The dashed curve corresponds to the unique prediction of tree-level χ PT, while the shaded region is the fit to the results of the NPLQCD calculations [21]. The NPLQCD calculations are performed at a single lattice spacing of $b \sim 0.125$ fm.

3 Hadron Scattering from Lattice QCD

To circumvent the Maiani-Testa theorem [17], which states that one cannot compute Green functions at infinite volume on the lattice and recover S-matrix elements except at kinematic thresholds, one computes the energy-eigenstates of the two particle system at finite volume to extract the scattering amplitude [18]. The scattering amplitude of two pions in the $I = 2$ -channel has been the process of choice to explore this technique, and to determine the reliability and systematics of lattice calculations.

3.1 $\pi\pi$ Scattering

NPLQCD has calculated $\pi\pi$ scattering in the $I = 2$ channel at relatively low pion masses using the mixed-action technique of LHPC. Domain-wall valence propagators are calculated on MILC configurations containing staggered sea-quarks. The lattice calculations were performed with the *Chroma* software suite [19,20] on the high-performance computing systems at the Jefferson Laboratory (JLab).

This calculation demonstrates the predictive capabilities of lattice QCD combined with low-energy EFT's. By writing the expansion of $m_\pi a_2$ as a function of m_π^2/f_π^2 , it has been shown that when inserting the values of m_π and f_π as calculated on the lattice. The lattice spacing effects in this mixed action calculation, which naively appear at $\mathcal{O}(b^2)$, are further suppressed, appearing only at higher orders [22,23]. The first non-trivial contributions from the partial-quenching in this calculation are shown to be numerically very small, and therefore a clean extraction of the counterterm in the χ PT Lagrangian is possible. The chiral extrapolation, as shown in Fig. 2, is found to have a smaller uncertainty at the physical point than the experimental value.

3.2 $K\pi$ Scattering

Studying the low-energy interactions between kaons and pions with $K^+\pi^-$ bound-states allows for an explicit exploration of the three-flavor structure of low-energy hadronic interactions, an aspect that is not directly probed in $\pi\pi$ scattering. Experiments have been proposed by the DIRAC collaboration [24] to study $K\pi$ atoms at CERN, J-PARC and GSI, the results of which would provide direct measurements or constraints on combinations of the scattering lengths. In the isospin limit, there are two isospin channels available to the $K\pi$ system, $I = \frac{1}{2}$ or $I = \frac{3}{2}$. The width of a $K^+\pi^-$ atom depends upon the difference between scattering lengths in the two channels, $\Gamma \sim (a_{1/2} - a_{3/2})^2$, (where $a_{1/2}$ and $a_{3/2}$ are the $I = \frac{1}{2}$ and $I = \frac{3}{2}$ scattering lengths, respectively) while the shift of the ground-state depends upon a different combination, $\Delta E_0 \sim 2a_{1/2} + a_{3/2}$.

The NPLQCD collaboration calculated the $K^+\pi^+$ scattering length, $a_{K^+\pi^+}$, in the same way as we computed $\pi\pi$ scattering, requiring only the additional generation of strange quark valence propagators [25]. As $a_{K^+\pi^+}$ was calculated (at a single lattice spacing of $b \sim 0.125$ fm) at three different pion masses, a chiral extrapolation was performed. This extrapolation depends upon two counterterms, one from the crossing-even and one from the crossing-odd amplitudes, and the important point is that their coefficients have different dependence upon the meson masses. Therefore, both can be determined from the results of the lattice calculation, as shown in Fig. 3, and therefore, the scattering lengths in **both** isospin channels can be predicted, as shown in Fig. 3. This is another demonstration of the combined power of lattice QCD and χ PT.

3.3 Nucleon-Nucleon Scattering

A few years ago we realized [26,27] that even though the scattering lengths in the nucleon-nucleon system are unnaturally large, and much larger than the spatial dimensions of currently available lattices, rigorous calculations in the NN-sector could be performed today. There are two aspects to this. First, it is unlikely that the scattering lengths in the NN sector are unnaturally large when computed on lattices with the lightest pion masses that are presently available, $m_\pi^{\text{sea}} \gtrsim 250$ MeV. Second, it is not the scattering length that dictates the lattice volumes that can be used in a Luscher-type analysis, but it is the range of the interaction, which is set by m_π for the NN interaction. While the Luscher asymptotic formula are not applicable when the scattering length becomes comparable to the spatial dimensions, the complete relation is still applicable [18],

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S} \left(\frac{pL}{2\pi} \right)$$

$$\mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{\|\mathbf{j}\| < \Lambda} \frac{1}{\|\mathbf{j}\|^2 - \eta^2} - 4\pi\Lambda. \quad (2)$$

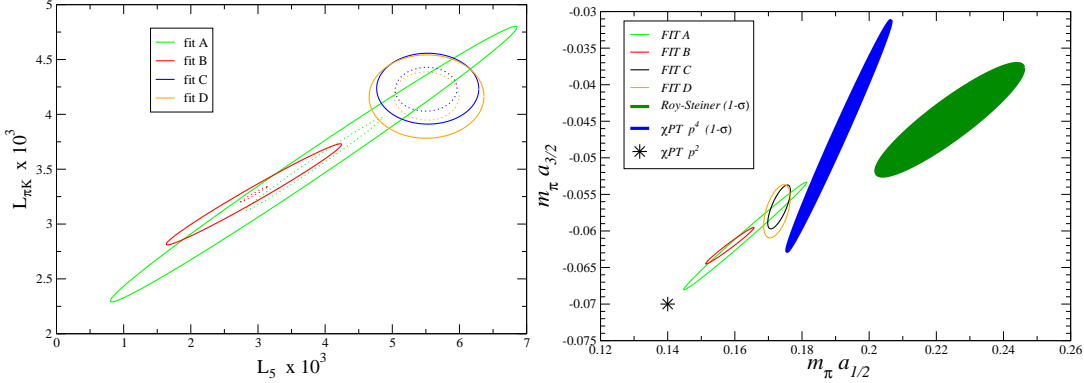


Fig. 3. The left panel shows the 95% confidence ellipses for the counterterms L_5 and $L_{K\pi}$ that contribute to $K\pi$ scattering at NLO in χ PT as extracted from the mixed-action lattice QCD calculation of NPLQCD at a lattice spacing of $b \sim 0.125$ fm [25]. The right panel shows the 95% confidence ellipses for the $K\pi$ scattering lengths from a combination of a lattice QCD calculation and χ PT. Also shown are the 38% confidence ellipses from a Roy-Steiner analysis and from a $\mathcal{O}(p^4)$ χ PT analysis.

where $\delta(p)$ is the elastic-scattering phase shift evaluated at kinetic energy $T = 2 \left(\sqrt{p^2 + M_N^2} - M_N \right)$, and the sum in eq. (2) is over all triplets of integers \mathbf{j} such that $|\mathbf{j}| < \Lambda$ and the limit $\Lambda \rightarrow \infty$ is implicit [27].

The same set of configurations that was used to produce the scattering lengths for $\pi\pi$ and $K\pi$ scattering, discussed above, were used to construct the NN correlation functions, and hence to extract the energy shifts between two nucleons at finite volume and twice the single nucleon mass at finite volume. At the pion masses accessible, the NN scattering lengths are found to be of natural size, set by the inverse pion mass. Only one of the pion masses is within the region described by the low-energy EFT's, and as such a prediction of the scattering length from lattice QCD alone cannot be made. However, when combined with the physical values of the scattering lengths, an allowed region for the scattering lengths as a function of pion mass can be made, as shown in Fig. 4.

3.4 Hyperon-Nucleon Scattering

Computation of hyperon-nucleon (and hyperon-hyperon) scattering amplitudes with lattice QCD is performed in the same way as the computation of nucleon-nucleon scattering amplitudes. The required strange quark propagator is not computationally expensive compared with the light-quark propagators, and so the calculation of these amplitudes costs very little more than that of the nucleon-nucleon amplitudes. NPLQCD has computed the correlation functions for hyperon-nucleon scattering in the channels with interpolating fields ΛN (spin $s = 0$ and $s = 1$), $\Sigma^- n$ (spin $s = 0$ and $s = 1$), $\Sigma^- \Sigma^-$, $\Lambda \Lambda$, $\Xi^- \Xi^-$ and is currently analyzing them in order to extract scattering phase-shifts for $m_\pi \sim 290, 350, 500$ and 600 MeV. In fig. 5 we show the preliminary correlation function and the effective scattering length for $\Sigma^- n$ in the spin-singlet channel for $m_\pi \sim 500$ MeV and at a lattice spacing of

$b \sim 0.125$ fm. Our preliminary analysis of these quantities gives a phase-shift of $\delta = -61.8 \pm 7.2^\circ$ at an energy of $T = 26.9 \pm 7.2$ MeV which would give a scattering length of $a = +0.61 \pm 0.12$ fm for an energy-independent $k \cot \delta$.

4 Hadronic Potentials

Lattice QCD cannot, in general, isolate the potential between hadrons, as the potential by itself is not an observable quantity. An exception to this statement is when both hadrons are infinite massive, and consequently their spatial separation can be fixed (this of course generalizes to N-hadrons). Currently, Silas Beane, William Detmold, Kostas Orginos and I are performing a quenched calculation of the potential between two B-mesons in the heavy quark limit. In this limit the light degrees of freedom (dof) of each B-meson decouple from the heavy-quark dynamics, and their spin, s_l , becomes a good quantum number. The potential between the two B-mesons when constructed from χ PT has the same form as that for NN, but with different values of the counterterms that enter. Therefore, calculating the potential between two B-mesons may provide qualitative insight into the NN-system. Our calculation is not the first attempt to extract the potential between heavy hadrons, but is performed with a lattice spacing approximately half of that of the previous calculations [34, 35]. The quark mass was chosen to give $m_\pi \sim 420$ MeV and $m_\rho \sim 700$ MeV.

Fig. 6 shows our preliminary results for the central potentials as a function of B-meson separation (the tensor potential is found to be very small, and so we have also shown the potentials at $r = \sqrt{2}$ and $\sqrt{5}$ which are not separated into tensor and central). One clearly sees short-distance repulsion in the channel with the quantum numbers of s-wave NN interactions, while one finds large attraction in the other channels. There is a correction due to the finite lattice spacing that is to be added to the results of the lattice calculation, and the leading correction

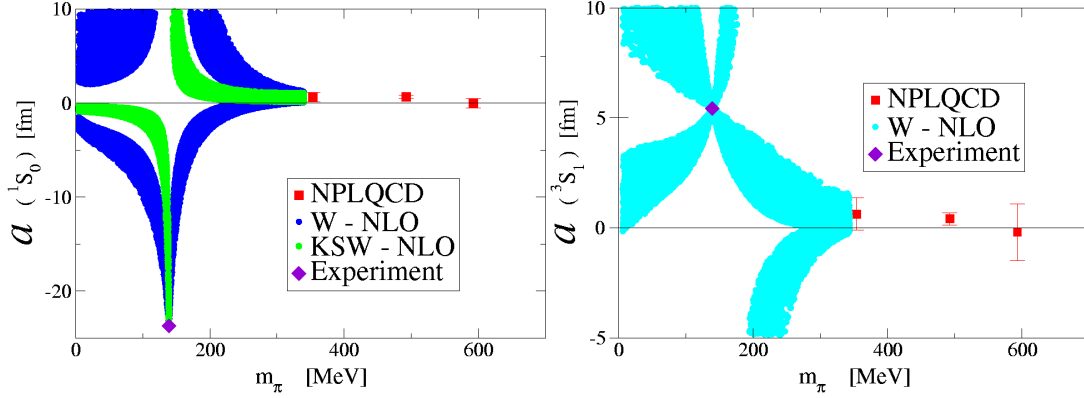


Fig. 4. Scattering lengths in the 1S_0 and $^3S_1 - ^3D_1$ NN channels as a function of pion mass. The experimental value of the scattering length and NDA have been used to constrain the extrapolation in both BBSvK [28,29,30] and W [31,32,33] power-countings at NLO.

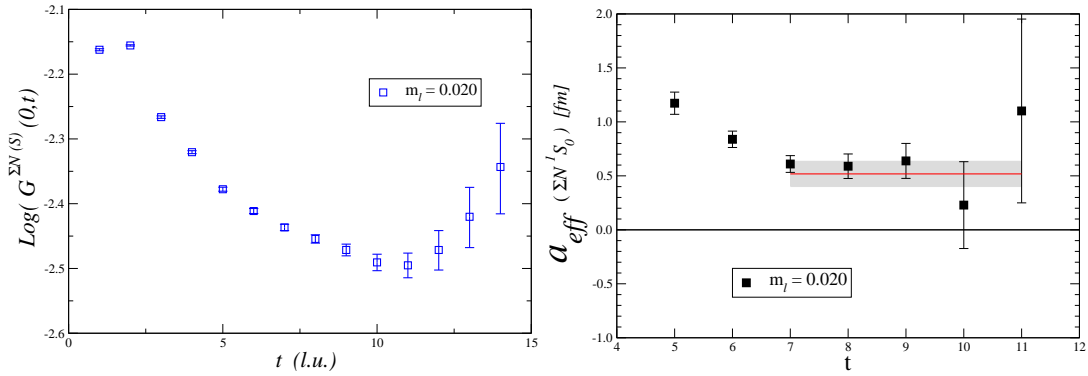


Fig. 5. The correlation function (left panel) and the effective scattering length plot (right panel) for neutron- Σ^- obtained in the coarse MILC lattices with $L \sim 2.5$ fm and $m_\pi \sim 500$ MeV.

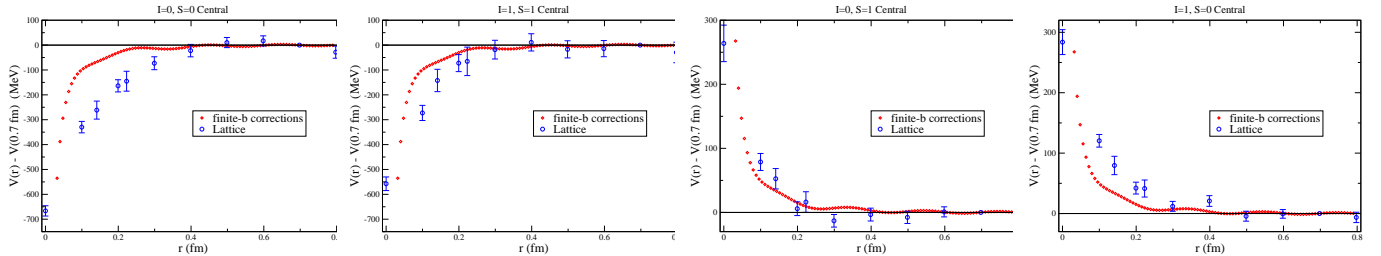


Fig. 6. Preliminary calculations of the potential between two B-mesons in the heavy quark limit. The vertical axis is the difference between the central potential at a separation r and that at $r = 7b$. The fine (red) dots correspond to the leading finite-lattice spacing correction that must be added to the potential obtained in the lattice calculation.

is shown in Fig. 6. Adding this factor gives the continuum potential between two infinitely massive B-mesons in the lattice volume, while the extrapolation to infinite volume potential has not yet been performed. The tensor interaction between hadrons has not been directly isolated in

previous works. In Fig. 7 we show our preliminary calculation of the tensor potential between two infinitely massive B-mesons. We find a potential that is consistent with zero within the uncertainty of the calculation, indicating that the tensor potential in this system is $V_T \lesssim 40$ MeV.

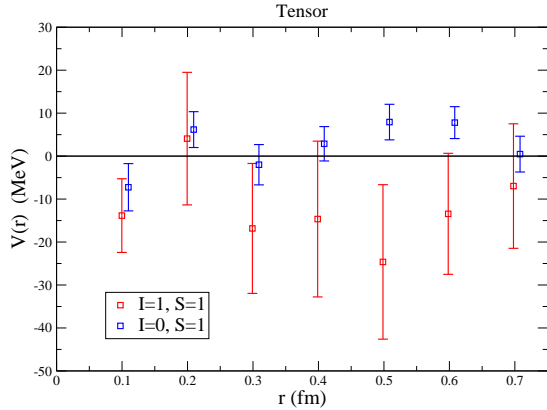


Fig. 7. Preliminary calculations of the tensor potential between two B-mesons in the heavy quark limit.

5 Outlook

It is clear that lattice QCD is an important part of the future of nuclear physics, and perhaps is an even more important part of the future of hypernuclear physics where there is significantly less experimental data to guide theoretical constructions. Lattice QCD, when combined with effective field theory, is just starting to make rigorous predictions of few-body observables, and we can expect significantly more progress as the computational resources dedicated to these calculations is increased. The formal tools are in place to explore the two-nucleon sector, and are being put in place to explore the hyperon-nucleon sector [36,26,37], all that is required is computational power. However, a coherent effort involving both numerical calculations and formal developments is presently required to move beyond the two-body sector and to explore hypernuclei.

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